# A demonstration apparatus for an acoustic analog to the Casimir effect†

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In the Casimir effect [1], two closely spaced uncharged parallel conducting plates mutually attract because their presence changes the mode structure of the quantum electromagnetic zero point field (ZPF) relative to free space. If the plates are a distance d apart, the force per unit area is $f = \hbar c/240d^4$, where $\hbar$ is the reduced Planck's constant and c is the speed of light in vacuum. Recently, Lamoreaux [2] has provided conclusive experimental verification of the Casimir force. The force between the two parallel plates can be understood in terms of the radiation pressure exerted by the plane waves that comprise the homogeneous, isotropic ZPF spectrum. In the space between the conducting plates, the modes formed by reflections off the plates act to push the plates apart. The modes outside the cavity formed by the plates act to push the plates together. The difference between the total outward pressure and the total inward pressure is the Casimir force per unit area.

Because the ZPF can be thought of as broadband noise, we can use broadband acoustic noise as an *analog* to the ZPF. In the acoustic Casimir effect, in contrast to the ZPF Casimir effect, broadband acoustic noise outside two parallel rigid plates drives the discrete modes between the plates. While external drivers can provide a steady state noise spectrum from which we can infer the energy per mode by dividing by the density of states $\omega^2/2\pi^2c^3$, this energy may be different in the cavity formed by the plates as a result of Q amplification. However, for this open resonant cavity the quality factor is poor, so we may assume it to be equal to unity, which renders the energy per mode equal to its value in free space.

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The radiation pressure of a wave with intensity $I$ incident at angle $\theta$ on a rigid plate is [3]

$$P = 2I\cos^2\theta/c \quad . \qquad (1)$$

For an isotropic noise spectrum with spectral intensity $I_\omega$ (measured by a microphone), the spectral intensity in the wave vector space of traveling waves is $I_{\mathbf{k}}=cI_\omega/4\pi k^2$, where the wave vector **k** has magnitude $k = \omega/c$. We choose the z axis to be normal to the plate, so that $k_z=k\cos\theta$. Thus, the total radiation pressure due to waves that strike the plate is

$$P_{out} = \frac{2}{c}\int dk_x dk_y dk_z I_{\mathbf{k}} \frac{k_z^2}{k^2} \quad , \qquad (2)$$

where the integration is over **k** values corresponding to waves that strike the plate.

Regarding the discrete modes between the plates, for convenience we continue to deal with the traveling wave modes. We label these modes with wavevector components $k_x=n_x\pi/L_x$, $k_y = n_y\pi/L_y$, and $k_z = n_z\pi/L_z$, where $n_x$, $n_y$, and $n_z$ are signed integers and $L_x$, $L_y$, and $L_z$ are the dimensions between the plates. We assume that the dimensions $L_x$ and $L_y$ of the plates are sufficiently large that the corresponding components of the wave vector are essentially continuous. Thus, in comparison to Eq. (2) the total inside pressure is [4]

$$P_{in} = \frac{2}{c}\sum \Delta k_z \int dk_x dk_y I_k \frac{k_z^2}{k^2} \quad , \qquad (3)$$

where $\Delta k_z = \pi/L_z$ and the sum is over values $n_z = n > 0$. The difference $P_{in}–P_{out}$ is the force per unit area between the plates, which is a continuous and piecewise differentiable function of the separation distance between the plates. The band limited noise can cause the force to be *attractive* or *repulsive* as a function of separation between the plates. On the other hand, if the lower frequency in the band is zero, the force is always attractive.

The repulsive force can be understood as follows. When the distance between the plates is comparable to the half wavelength associated with the lower edge of the frequency band, the corresponding modes inside the plates have wavevectors that are nearly perpendicular to the plates. However, the modes outside the plates corresponding to the same frequencies are spread over all possible angles of incidence. Thus for the same total intensity, the momentum transfer due to waves inside the plates is over a narrow cone while the momentum transfer due to waves outside the plates extends over all angles, leading to a repulsive force. We performed a quantitative measurement of the force between two plates due to the radiation pressure of broadband acoustic noise. The results, shown in Figure 1, and the details of the experiment have been reported in Ref. 4.

To demonstrate the force between two rigid plates due to broadband acoustic noise, we have constructed the apparatus shown in Fig 2. When placed in an enclosure and exposed to isotropic and homogeneous acoustic noise of sufficient intensity, the plates are seen to attract for small separation distances. The attraction is strongest for separation distances smaller that one half–wavelength of the upper frequency of the noise. Repulsion can be seen for plate separation distances of about one half–wavelength of the lower frequency of the noise. The acoustic analog to the Casimir effect has been successfully demonstrated to a wide variety of audiences, from senior physics majors to attendees of scientific meetings [5]. The effect is not subtle and can be demonstrated in a large lecture hall.

The acoustic Casimir demonstration device is a 1/4-inch cylindrical aluminum rod 12 inch long with a 4 inch square pane attached to each end. Each pane is a 1/16–inch thick aluminum plate (Fig 2a). The rod is pivoted at the center with a V-jewel bearing and a steel pin [6]. Two additional aluminum plates 4 inch square and 1/16–inch thick are mounted on rigid stands and positioned to oppose the plates mounted on the rod, in a configuration reminiscent of Cavendish's balance (Fig 2b). The device is placed in a clear acrylic box of inside dimensions 16×16×6 inch and thickness 1 inch (Fig. 3). Two JBL 2402 ultra-high frequency compression drivers are employed to generate the noise field [7]. These were chosen for their 2.5 – 15 kHz response range and, more importantly, their sensitivity of 110 dB (re: $10^{-12}$ W/m$^2$) for 1.0 W at 1.0 m. The full response range can be utilized and the drivers could safely output 135 dB in the enclosure. The lower frequency of 2.5 kHz is well above the lower mode of the box (~400 Hz) providing a fairly homogeneous noise distribution within the box. Because the acrylic box is 1 inch thick, the acoustic radiation outside the box is small and no ear protection is necessary.

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[1] H. B. G. Casimir, "On the attraction between two perfectly conducting plates," *Proc. Kon. Ned. Akad. Wetensch*. **51**, 793-796 (1948). E. Elizalde and A. Romeo, " Essentials of the Casimir effect and its computation," *Am. J. Phys*. **59**, 711–719 (1991). V. Hushwater, " Repulsive Casimir force as result of vacuum radiation pressure," *Am. J. Phys.* **65**, 381-384 (1997).

[2] S. K. Lamoreaux, "Demonstration of the Casimir force in the 0.6 to 6 μm range," *Phys. Rev. Lett*. **78**, 5-8 (1997).

[3] L. Landau and E. Lifshitz, *Fluid Mechanics* (Pergamon Press, London, 1959), Sec. 65, pp. 255-256, considered an acoustic plane wave incident at an arbitrary angle at the interface of two fluids. The radiation pressure at a rigid interface is obtained in the limit when the impedance (the product of the density times the speed of sound) of one of the fluids is infinite.

[4] A. Larraza, C. Holmes, R. Susbilla, and B. Denardo, "The force between two parallel rigid plates due to the radiation pressure of broadband noise: An acoustic Casimir effect," *J. Acoust. Soc. Am*. **103**, 2267-2272 (1998). A. Larraza and Bruce Denardo, "An acoustic Casimir effect," *Phys. Lett. A* **248**, 151-155 (1998).

[5] A. Larraza, "Some acoustic analogs to electromagnetic zero point field effects: Static and dynamic Casimir effects," *Advanced ICFA beam dynamics workshop on "Quantum aspects of beam physics,"* January 4–9, Monterey, CA (1998).

[6] Small Parts, Inc. (Miami Lakes, Florida). Telephone number: 800–220–4242. The part numbers for the V–jewel and steel pin are: R–VJ–1244 (V–jewel bearing) and VJPX–7D (steel pin).

[7] JBL Professional (Northridge, California). Telephone number: 818–894–8850. The price of a JBL 2402H driver is about $250.

## FIGURE CAPTIONS

**Figure 1** Force between two parallel rigid plates 15 cm in diameter as a function of the distance between them. The points are experimental data, and the curves are from theory with no adjustable parameters for (a) a flat spectrum and (b) a piecewise power–law spectrum. The spectrum is in a band of frequencies between 4.8 – 16 kHz. The total intensity of the noise is 133 dB (re $10^{-12}$ W/m$^2$). As a result of the finite lower cutoff spectral frequency, the force can be repulsive at some separation distances between the plates. The details of the experiment have been reported in Ref. 4.

**Figure 2** The acoustic Casimir demonstration device consists of (a) a 1/4-inch cylindrical aluminum rod 12 inch long with a 4 inch square pane attached to each end. Each pane is a 1/16–inch thick aluminum plate. The rod is pivoted at the center with a V-jewel bearing and a steel pin (Ref. 6). (b) Two additional aluminum plates 4 inch square and 1/16–inch thick are mounted on rigid stands and positioned to oppose the plates mounted on the rod, in a configuration reminiscent of Cavendish's balance. When placed in an enclosure and exposed to isotropic and homogeneous acoustic noise of sufficient intensity, the plates free to rotate are attracted or repelled to the plates mounted on stands. Attraction occurs for small separation distances, while repulsion can be seen for plate separation distances of about one half–wavelength of the lower cutoff frequency.

**Figure 3** Photograph of the device described in Fig 2 inside a clear acrylic box of inside dimensions 16×16×6 inch and thickness 1 inch. Two JBL 2402 ultra-high frequency compression drivers (Ref. 7) are employed to generate the noise field. The drivers are mounted flush with the floor at two opposite corners of the acrylic box. The full response range (2.5 kHz –15 kHz) can be utilized and the JBL 2402 drivers could safely output 135 dB in the enclosure. Because the acrylic used for the box is 1 inch thick, the acoustic radiation outside the box is small and no ear protection is necessary.

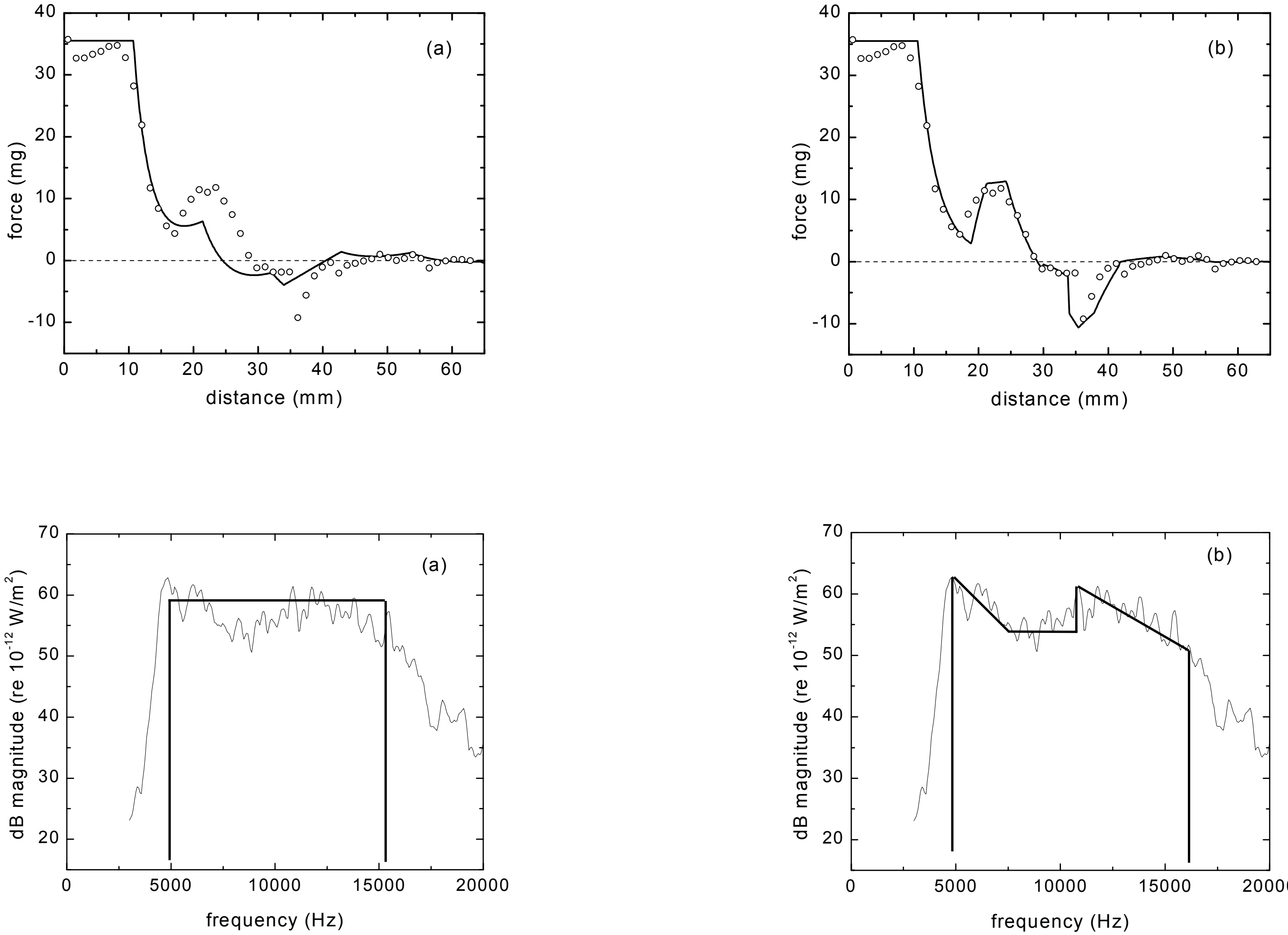

(a)
(b)
force (mg)
distance (mm)
dB magnitude (re 10^-12 W/m^2)
frequency (Hz)

**FIGURE 1**

(a)

(b)

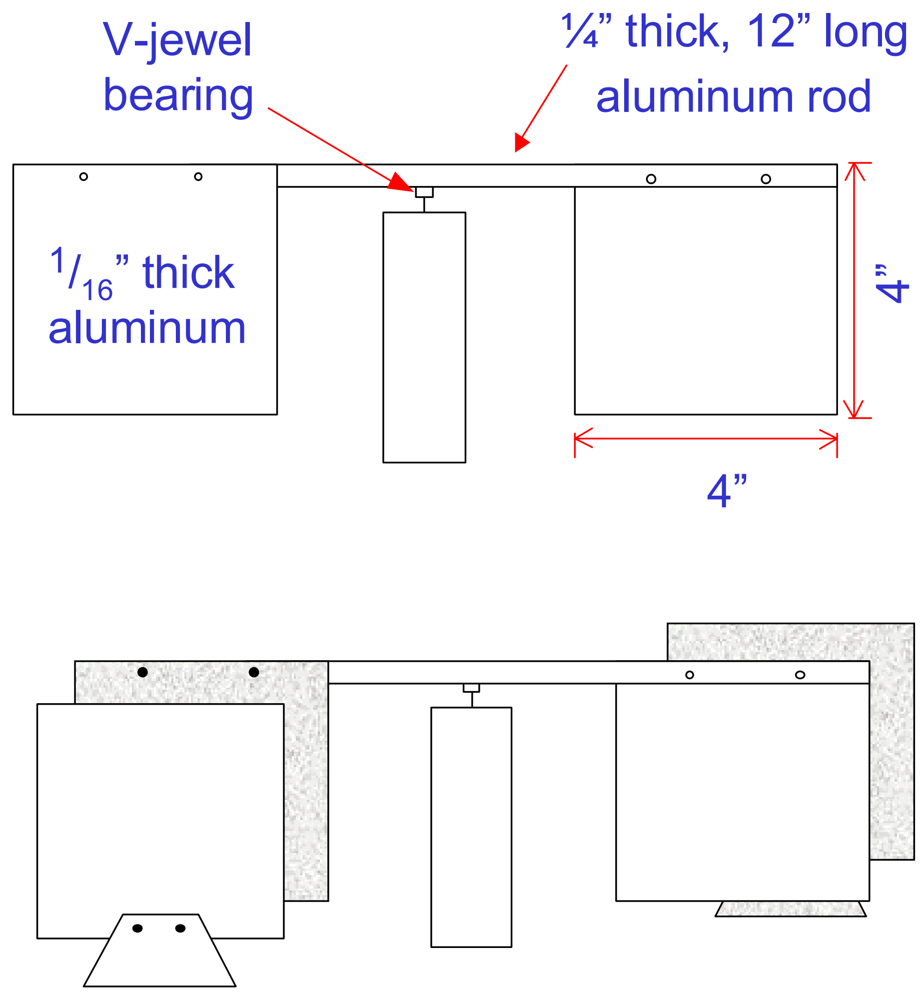
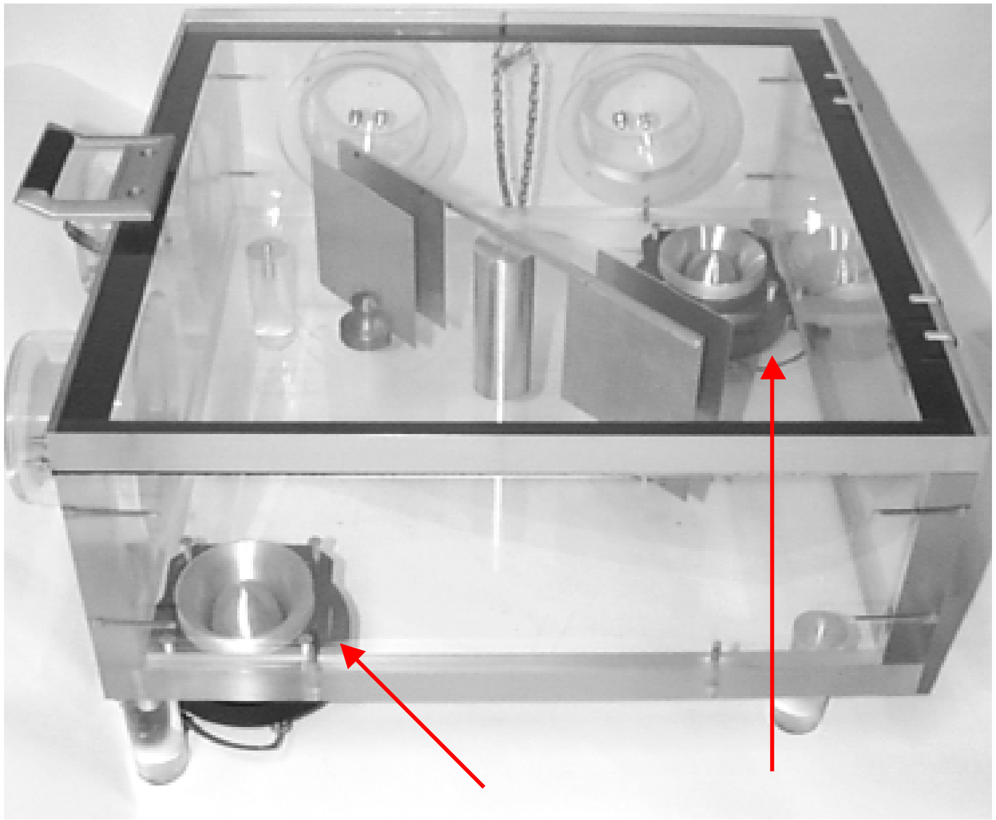

V-jewel bearing
¼” thick, 12” long aluminum rod
1/16” thick aluminum
4”
4”

**FIGURE 2**



**JBL 2402 compression drivers**

**FIGURE 3**