1) Calculate the standoff distance for the magnetopause for the conditions encountered during shocks, or periods of high speed solar wind:
$v=8\times10^5$ m/s, $n = 25\times10^6\ m^{-3}$

$$\left(\frac{r}{R_\oplus}\right)_s = \left(\frac{4\,B_{os}^2}{2\mu_o n m v^2}\right)^{1/6} = \left(\frac{4\times\left(3\times10^{-5}\right)^2}{\left(8\pi\times10^{-7}\right)\left(2.5\times10^{7}\right)\left(1.67\times10^{-27}\right)\left(8\times10^{5}\right)^2}\right)^{1/6} = 6.1$$

2) What is the co-rotation electric field? In cylindrical coordinates we have:

$$\vec{\mathbf{U}} = \omega\, r\, \hat{\phi}, \quad \mathbf{B} = B_o\left(\frac{R_\oplus}{r}\right)^3\left(-\hat{\theta}\right),$$

where U is the streaming, or drift velocity.

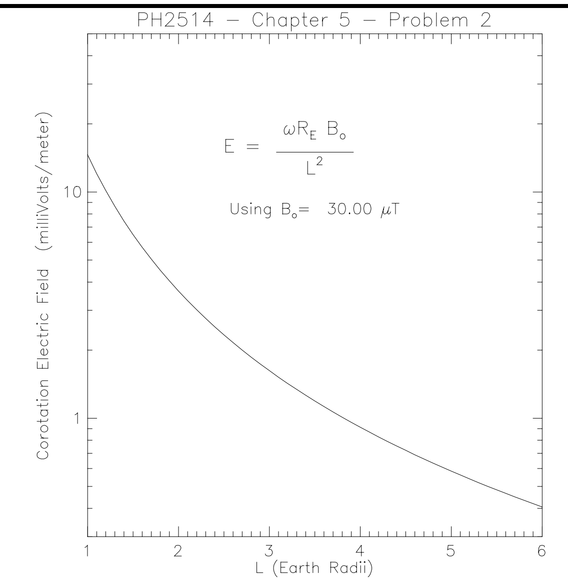
The frozen-in-field condition gives: $\mathbf{E} = -\mathbf{U}\times\mathbf{B}$

This velocity will solve the drift relation: $\vec{\mathbf{U}} = \dfrac{\vec{\mathbf{E}}\times\vec{\mathbf{B}}}{B^2}$, as can be seen...

$$\vec{\mathbf{U}} = \frac{\vec{\mathbf{E}}\times\vec{\mathbf{B}}}{B^2} = -\frac{\left(\mathbf{U}\times\mathbf{B}\right)\times\mathbf{B}}{B^2}$$
$$= \frac{\mathbf{U}\left(\mathbf{B}\bullet\mathbf{B}\right)}{B^2} - \frac{\mathbf{B}\left(\mathbf{U}\bullet\mathbf{B}\right)}{B^2}$$

the latter step is a vector identity. The first term is just $\mathbf{U}$, the second is zero since $\mathbf{U}\perp\mathbf{B}$. Now

$$\mathbf{E}_{corotation} = -\mathbf{U}\times\mathbf{B}$$
$$= \omega\, r\, \hat{\phi} \times B_o\left(\frac{R_\oplus}{r}\right)^3\left(\hat{\theta}\right) = \frac{\omega B_o R^3}{r^2}\left(-\hat{\mathbf{r}}\right)$$



3) Calculate the invariant latitudes corresponding to L = 4 and L = 6.6

$$\cos\Lambda_c = \frac{1}{\sqrt{L}};$$

$$L = \begin{Bmatrix}4\\6.6\end{Bmatrix} \quad \cos\Lambda_c = \begin{Bmatrix}0.5\\0.39\end{Bmatrix} \quad \Lambda_c = \begin{Bmatrix}60^\circ\\67^\circ\end{Bmatrix}$$

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4) $n\left(\mathrm{cm}^{-3}\right) = 100\left(\frac{4.5}{L}\right)^{4}$

L	2	3	4	5	6	7	8	9	10
n	2563	506	160	66	31.6	17.1	10.0	6.25	4.10

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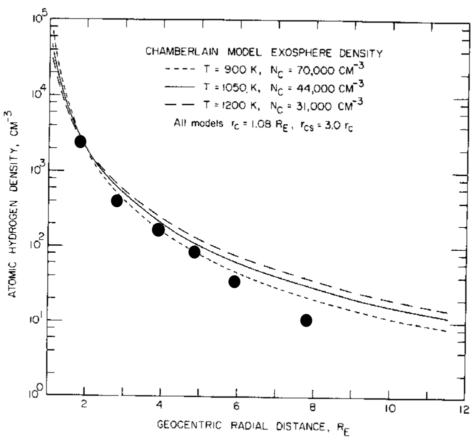

Exospheric hydrogen density versus radial distance for the Chamberlain models of Figure 5. The model a temperature $T = 1050$ K provides the best fit to the DE 1 geocoronal observations.

5.

$$\vec{v}_D = \frac{K_\perp}{qB}\frac{\vec{\mathbf{B}}\times\nabla B}{B^2} \quad ; \quad \nabla B = -\frac{3}{r}B\,\hat{\mathbf{r}}\,(\lambda=0); \quad \left.\frac{\vec{\mathbf{B}}\times\nabla B}{B^2}\right|_{\lambda=0} = -\frac{3}{r}\ -\hat{\theta}\times\hat{\mathbf{r}} = -\frac{3}{r}\hat{\phi}$$

$$\vec{v}_D = -\frac{3}{r}\frac{K_\perp}{qB}\hat{\phi}$$

$$\left|v_D\right| = \frac{3}{r}\frac{K_\perp}{qB} = \frac{3}{LR_\oplus}\left(\frac{K_\perp}{q}\right)\frac{K_\perp}{q}\frac{1}{B}.$$ Here, the term $\frac{K_\perp}{q}$ can be taken as the energy in eV

$$\left|v_D\right| = \frac{3}{r}\frac{K_\perp}{qB} = \frac{3}{LR_\oplus}\left(\frac{K_\perp}{q}\right)\frac{L^3}{B_o} = \frac{3\,L^2}{R_\oplus B_o}\left(\frac{K_\perp}{q}\right)$$

$$R_\oplus = 6.37\times10^6\ \text{m},\ B_o = 3\times10^{-5}\ \text{T},\ L=3,\ \frac{K_\perp}{q} = 10^6\ \text{eV}$$

$$\left|v_D\right| = \frac{3\bullet3^2}{6.37\times10^6\bullet3\times10^{-5}}10^6 = \frac{9}{6.37}\times10^5 = 1.41\times10^5\ \ {}^{m}\!/_{s}$$

Compare this to the thermal velocity for a proton..

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{3.2\times10^{-13}}{1.67\times10^{-27}}} = 1.38\times10^7 \ \ldots$$ much larger than the drift speed

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6) Estimate the density for ring current ions at L = 3.
we have from the problem statement...

$$\text{flux} = 3\times 10^{11} \ \frac{\text{ions}}{\text{m}^2\ \text{s}} = \text{nv}.$$ From above, we have a thermal velocity of

$v = 1.38\times 10^7 \ {}^{\text{m}}\!/\!_{\text{s}}$. Dividing out gives $n = 2.2\times 10^4 \ {}^{\text{ions}}\!/\!_{\text{m}^3}$

about 6 orders of magnitude down from the background, 0.5 eV, plasmasphere ions, or even the 0.1 eV neutral hydrogen.

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7) Estimate the ring current which must result from the above density, and drift velocity.

$$\text{flux} = \text{nv} = 2.2\times 10^4 \ {}^{\text{ions}}\!/\!_{\text{m}^3} \bullet 1.41\times 10^5 \ {}^{\text{m}}\!/\!_{\text{s}} = 3.1\times 10^9 \ {}^{\text{ions}}\!/\!_{\text{m}^2\ \text{s}}$$

$$\text{current density} = \text{qnv} = 1.6\times 10^{-19} \bullet 3.1\times 10^9 \ {}^{\text{ions}}\!/\!_{\text{m}^2\ \text{s}} = 4.9\times 10^{-10} \ {}^{\text{Amperes}}\!/\!_{\text{m}^2}$$

$$I = J\,A = 4.9\times 10^{-10} \bullet \left(6.37 \ \times 10^6\right)^2 = 2\times 10^4 \ \text{Amperes}$$

Compare this to the current necessary to produce a 100 nT change in B, assuming a current loop. From Halliday and Resnick (page 863)

$$B = \frac{\mu_o\ i}{2\ R} = 2\pi\times 10^{-7} \ \frac{2\times 10^4}{3\bullet 6.37\times 10^6} = 6.6\times 10^{-10} \ \text{Tesla} = 0.6 \text{ nano - Tesla}$$

Apparently, we need more ions......

---

8)

$$\oint \mathbf{E}\bullet \mathbf{dl} = -\ \frac{d\Phi_B}{dt}$$

$$E \bullet 2\pi\ r = -\pi r^2 \frac{dB}{dt} \Rightarrow E = -\ \frac{r}{2}\frac{dB}{dt} = -\frac{20\times 10^6}{2}\frac{10^{-7}}{100 \text{ to } 1000}$$

$$= -10^{-3} \ \text{ to } 10^{-2} \quad \text{Volts / meter}$$

If this field extends all the way around (10^8 m), the total potential around the earth is 0.1 to 1 million volts

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9) We have two equations to consider, from chapters 4 and 5 respectively

$$B = B_0 R_\oplus^3 \frac{\sqrt{1+3\sin^2\lambda}}{r^3} = \frac{B_0\sqrt{1+3\sin^2\lambda}}{(r/R_\oplus)^3} \quad \text{(eqn 4.5)}$$

$$B = \frac{B_{os}}{L^3}\frac{\sqrt{4 - 3\cos^2\lambda_m}}{\cos^6\lambda_m} \quad \text{(eqn 5.21)}$$

The latter is the one to use....Take: $B_{os} = 3.0 \times 10^{-5}$, although it will factor out...
L = 5, λ= 0, 45

$$B = \frac{3.0 \times 10^{-5}}{5^3}\frac{\sqrt{4 - 3\cos^2\lambda_m}}{\cos^6\lambda_m} = 2.4 \times 10^{-7}\frac{\sqrt{4 - 3\cos^2\lambda_m}}{\cos^6\lambda_m}$$

$$B = 2.4 \times 10^{-7}\begin{cases} 1 & \lambda_m = 0 \\ \dfrac{\sqrt{4 - 3 \bullet 0.5}}{0.5^3} & \lambda_m = 45 \end{cases} = 2.4 \times 10^{-7}\begin{cases} 1.00 & \lambda_m = 0 \\ 12.65 & \lambda_m = 45 \end{cases} = \begin{cases} 2.4 \times 10^{-7} & \lambda_m = 0 \\ 30.4 \times 10^{-7} & \lambda_m = 45 \end{cases}$$

The mirror formula is

$$\frac{\sin^2(\alpha)}{B} = \frac{\sin^2(\alpha_m)}{B_m} \Rightarrow \frac{\sin^2(\alpha_{critical})}{B} = \frac{\sin^2(90)}{B_m} \Rightarrow \sin^2(\alpha_{critical}) = \frac{B}{B_m} = \frac{1}{12.65}$$

$$\sin(\alpha_{critical}) = \frac{1}{3.56} \Rightarrow \quad \alpha_{critical} = 16.33^\circ$$

particles with pitch angles larger than this will mirror before reaching 45degrees magnetic latitude

11. Calculate the energies for which the corotation electric field induced drift velocity $\vec{\mathbf{U}} = \omega\, r\, \hat{\phi}$, equals the ∇B drift velocity, for protons at L = 3, and L = 6.6. For simplicity, work in the equatorial plane, and look only at the grad-B drift.

$$\vec{v}_D = -\frac{3}{r}\frac{K_\perp}{qB}\hat{\phi} = \omega r\ \hat{\phi} = \vec{v}_{corotation} \quad \Rightarrow$$

$$K_\perp = \omega r^2\frac{qB}{3} = L^2\omega R_{earth}^2\frac{qB_{os}\frac{1}{L^3}}{3} = \omega R_{earth}^2\frac{qB_{os}}{3L}$$

$$= \frac{2\pi}{86400}\left(6.38\times10^6\right)^2\frac{1.6\times10^{-19}\bullet 3\times10^{-5}}{3L} = \frac{2\pi}{86400}\left(6.38\times10^6\right)^2\frac{1.6\times10^{-19}\bullet 3\times10^{-5}}{3L}$$

$$= \frac{4.74\times10^{-15}}{L}\text{Joules} \quad = \frac{29{,}601}{L}\ \text{eV}$$

so, at L =3, the corotation velocity is equal and opposite to the grad-B drift for an energy of 9.9 keV; at L = 6.6, the two are equal for E=4.5 keV