1) Calculate the standoff distance for the magnetopause for the conditions encountered during shocks, or periods of high speed solar wind:
$v=8\times10^5$ m/s, $n = 25\times10^6$ m^{-3}

$$\left(\frac{r}{R_\oplus}\right)_s = \left(\frac{4\,B_{os}^2}{2\mu_o n m v^2}\right)^{1/6} = \left(\frac{4\times\left(3\times10^{-5}\right)^2}{\left(8\pi\times10^{-7}\right)\left(2.5\times10^7\right)\left(1.67\times10^{-27}\right)\left(8\times10^5\right)^2}\right)^{1/6} = 6.1$$

---

2) What is the co-rotation electric field? In cylindrical coordinates we have:

$$\vec{\mathbf{U}} = \omega r\,\hat{\boldsymbol{\phi}}, \quad \mathbf{B} = B_o\left(\frac{R_\oplus}{r}\right)^3\left(-\hat{\boldsymbol{\theta}}\right),$$

where U is the streaming, or drift velocity.

The frozen-in-field condition gives: $\mathbf{E} = -\mathbf{U}\times\mathbf{B}$

This velocity will solve the drift relation: $\vec{\mathbf{U}} = \dfrac{\vec{\mathbf{E}}\times\vec{\mathbf{B}}}{B^2}$. The proof is given here:

$$\vec{\mathbf{U}} = \frac{\vec{\mathbf{E}}\times\vec{\mathbf{B}}}{B^2} = -\frac{(\mathbf{U}\times\mathbf{B})\times\mathbf{B}}{B^2}$$

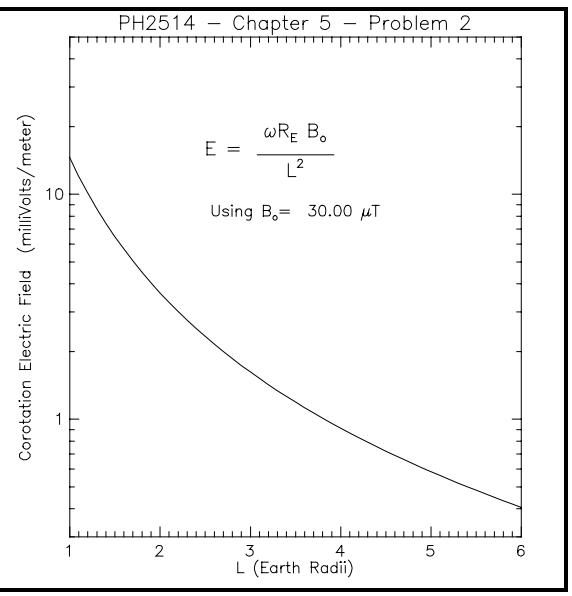
$$= \frac{\mathbf{U}(\mathbf{B}\bullet\mathbf{B})}{B^2} - \frac{\mathbf{B}(\mathbf{U}\bullet\mathbf{B})}{B^2}$$

the latter step is a vector identity. The first term is just $\mathbf{U}$, the second is zero since $\mathbf{U}\perp\mathbf{B}$.

Now

$$\mathbf{E}_{\text{corotation}} = -\mathbf{U}\times\mathbf{B}$$

$$= \omega r\,\hat{\boldsymbol{\phi}} \times B_o\left(\frac{R_\oplus}{r}\right)^3\left(\hat{\boldsymbol{\theta}}\right) = \frac{\omega B_o R^3}{r^2}\left(-\hat{\mathbf{r}}\right)$$



3) Calculate the invariant latitudes corresponding to L = 4 and L = 6.6

$$\cos\Lambda_c = \frac{1}{\sqrt{L}};$$

$$L = \left.\begin{matrix}4\\6.6\end{matrix}\right\} \quad \cos\Lambda_c = \left.\begin{matrix}0.5\\0.39\end{matrix}\right\} \quad \Lambda_c = \left.\begin{matrix}60^\circ\\67^\circ\end{matrix}\right\}$$

---

4) $n\left(cm^{-3}\right) = 100\left(\frac{4.5}{L}\right)^4$

L	2	3	4	5	6	7	8	9	10
n	2563	506	160	66	31.6	17.1	10.0	6.25	4.10

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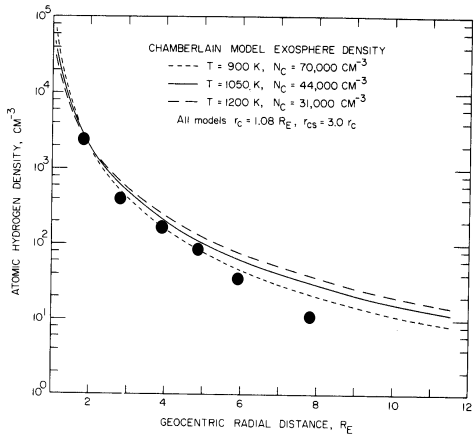

Exospheric hydrogen density versus radial distance for the Chamberlain models of Figure 5. The model at temperature T = 1050 K provides the best fit to the DE 1 geocoronal observations.

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5. Solve the problem at the magnetic equator: $\lambda_m = 0^\circ$

$$\vec{v}_D = \frac{K_\perp}{qB}\frac{\vec{\mathbf{B}}\times\nabla B}{B^2} \quad ; \quad \nabla B = -\frac{3}{r}\frac{B_{os}}{r^3}\,\hat{\mathbf{r}}\ (\lambda=0) = -\frac{3}{r}B\,\hat{\mathbf{r}}\ ; \quad \left.\frac{\vec{\mathbf{B}}\times\nabla B}{B^2}\right|_{\lambda=0} = -\frac{3}{r}\,\hat{\boldsymbol{\theta}}\times\hat{\mathbf{r}} = -\frac{3}{r}\hat{\boldsymbol{\phi}}$$

$$\vec{v}_D = -\frac{3}{r}\frac{K_\perp}{qB}\hat{\boldsymbol{\phi}}$$

$$|v_D| = \frac{3}{r}\frac{K_\perp}{qB} = \frac{3}{LR_\oplus}\left(\frac{K_\perp}{q}\right)\frac{K_\perp}{q}\frac{1}{B}.$$ Here, the term $\frac{K_\perp}{q}$ can be taken as the energy in eV

$$|v_D| = \frac{3}{r}\frac{K_\perp}{qB} = \frac{3}{LR_\oplus}\left(\frac{K_\perp}{q}\right)\frac{L^3}{B_o} = \frac{3\,L^2}{R_\oplus B_o}\left(\frac{K_\perp}{q}\right)$$

$R_\oplus = 6.37\times10^6$ m, $B_o = 3\times10^{-5}$ T, L=3, $\frac{K_\perp}{q} = 10^6$ eV

$$|v_D| = \frac{3\bullet 3^2}{6.37\times10^6\bullet 3\times10^{-5}}10^6 = \frac{9}{6.37}\times10^5 = 1.41\times10^5\ \ m/s$$

Compare this to the thermal velocity for a proton..

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{3.2\times10^{-13}}{1.67\times10^{-27}}} = 1.38\times10^7 \ \ldots$$ much larger than the drift speed

---

6) Estimate the density for ring current ions at L = 3.
we have from the problem statement...

flux $= 3\times10^{11}\ \frac{\text{ions}}{\text{m}^2\ \text{s}} = \text{nv}$. From above, we have a thermal velocity of $v = 1.38\times10^{7}\ \text{m/s}$. Dividing out gives $n = 2.2\times10^{4}\ \text{ions/m}^3$
about 6 orders of magnitude down from the background, 0.5 eV, plasmasphere ions, or even the 0.1 eV neutral hydrogen.

---

7) Estimate the ring current which must result from the above density, and drift velocity.

flux $= \text{nv} = 2.2\times10^{4}\ \text{ions/m}^3 \bullet 1.41\times10^{5}\ \text{m/s} = 3.1\times10^{9}\ \text{ions/m}^2\ \text{s}$

current density $= \text{qnv} = 1.6\times10^{-19} \bullet 3.1\times10^{9}\ \text{ions/m}^2\ \text{s} = 4.9\times10^{-10}\ \text{Amperes/m}^2$

$I = J\,A = 4.9\times10^{-10} \bullet \left(6.37\times10^{6}\right)^2 = 2\times10^{4}$ Amperes

Compare this to the current necessary to produce a 100 nT change in B, assuming a current loop. From Halliday and Resnick (page 863)

$$B = \frac{\mu_o\, i}{2\,R} = 2\pi\times10^{-7}\ \frac{2\times10^{4}}{3\bullet 6.37\times10^{6}} = 6.6\times10^{-10}\ \text{Tesla} = 0.6\ \text{nano-Tesla}$$

---

8) $\oint \mathbf{E}\bullet\mathbf{dl} = -\frac{d\Phi_B}{dt}$

$$E \bullet 2\pi\ r = -\pi\ r^2 \frac{dB}{dt} \Rightarrow E = -\frac{r}{2}\frac{dB}{dt} = -\frac{20\times10^{6}}{2}\ \frac{10^{-7}}{100\text{ to }1000}$$

$= -10^{-3}$ to 10^{-2} Volts/meter

If this field extends all the way around (10^8 m), the total potential around the earth is 0.1 to 1 million volts

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9) We have two equations to consider, from chapters 4 and 5 respectively

$$B = B_0 R_\oplus^3 \frac{\sqrt{1+3\sin^2\lambda}}{r^3} = \frac{B_0\sqrt{1+3\sin^2\lambda}}{\left(r/R_\oplus\right)^3} \quad \text{(eqn 4.5)}$$

$$B = \frac{B_{os}}{L^3}\,\frac{\sqrt{4 - 3\cos^2\lambda_m}}{\cos^6\lambda_m} \quad \text{(eqn 5.21)}$$

The latter is the one to use....Take: $B_{os} = 3.0 \times 10^{-5}$, although it will factor out…
L = 5, λ= 0, 45

$$B = \frac{3.0 \times 10^{-5}}{5^3}\,\frac{\sqrt{4 - 3\cos^2\lambda_m}}{\cos^6\lambda_m} = 2.4 \times 10^{-7}\,\frac{\sqrt{4 - 3\cos^2\lambda_m}}{\cos^6\lambda_m}$$

$$B = 2.4 \times 10^{-7}\begin{cases} 1 & \lambda_m = 0 \\ \dfrac{\sqrt{4 - 3\bullet 0.5}}{0.5^3} & \lambda_m = 45 \end{cases} = 2.4 \times 10^{-7}\begin{cases} 1.00 & \lambda_m = 0 \\ 12.65 & \lambda_m = 45 \end{cases} = \begin{cases} 2.4 \times 10^{-7} & \lambda_m = 0 \\ 30.4 \times 10^{-7} & \lambda_m = 45 \end{cases}$$

The mirror formula is

$$\frac{\sin^2(\alpha)}{B} = \frac{\sin^2(\alpha_m)}{B_m} \Rightarrow \frac{\sin^2(\alpha_{critical})}{B} = \frac{\sin^2(90)}{B_m} \Rightarrow \sin^2(\alpha_{critical}) = \frac{B}{B_m} = \frac{1}{12.65}$$

$$\sin(\alpha_{critical}) = \frac{1}{3.56} \Rightarrow \quad \alpha_{critical} = 16.33^\circ$$

particles with pitch angles larger than this will mirror before reaching 45degrees magnetic latitude.

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11. Calculate the energies for which the corotation electric field induced drift velocity $\vec{\mathbf{U}} = \omega r\,\hat{\boldsymbol{\phi}}$, equals the ∇B drift velocity, for protons at L = 3, and L = 6.6. For simplicity, work in the equatorial plane, and look only at the grad-B drift.

Start with: $\left|\vec{v}_D\right| = \frac{3}{r}\frac{K_\perp}{qB} = \left|\vec{\mathbf{U}}_{corotation}\right| = \omega r$

$$\frac{3}{r}\frac{K_\perp}{qB} = \omega r \Rightarrow K_\perp = \omega r \bullet \frac{r}{3}\frac{qB}{1} = q\frac{\omega r^2}{3}B$$

$$K_\perp = q\frac{\omega r^2}{3}B = q\frac{\omega r^2}{3}\frac{B_{os}}{L^3} = q\frac{\omega\left(L\bullet R_E\right)^2}{3}\frac{B_{os}}{L^3} = q\frac{\omega R_E^2}{3}\frac{B_{os}}{L}$$

$$= q\frac{2\pi\bullet 3.1\times10^{-5}\bullet\left(6.38\times10^6\right)^2}{3\bullet 86400}\frac{1}{L} = \frac{q}{L}\frac{7.93\times10^9}{259200} = \frac{q}{L}\bullet 30600$$

$$K_\perp(eV) = \frac{K_\perp(Joules)}{1.6\times10^{-19}} = \frac{1}{L}\bullet 30600$$

so – 10.2 keV at L = 3, 4.6 keV at L = 6.6